4/20/18 "I am not a product of my circumstances. I am a product of my decisions."-Stephen Covey

HW: "Geometric Sequences and Series" Finish the packet

AIM: What is a Geometric Sequence?

Warm Up:

$\times 3 \quad \times 3 \quad \times 3 \quad \times 3$

Given the sequence 3, 9, 27, 81, ... 243, 729, 2187, 6561, 19683, $\boxed{59049}$

(a) Find the tenth term.

10^{th} term = 59049

(b) Can you write a formula to help you find any given term of the sequence?

$$a_n = 3 \cdot 3^{n-1}$$

Starting Value a_1

number we multiply by over and over

FLUENCY

1. Which of the following represents the sum of $3+10+\cdots+87+94$ if the arithmetic series has 14 terms?

(1) 1,358 (3) 679

(2) 658 (4) 1,276

$$S_{14}=\frac{14}{2}(3+94)=7(97)=679$$

(3)

2. The sum of the first 50 natural numbers is

(1) 1,275 (3) 1,250

(2) 1,875 (4) 950

$$S_{50}=1+2+3+\cdots+49+50$$
$$=\frac{50}{2}(1+50)=25(51)=1275$$

(1)

3. If the first and last terms of an arithmetic series are 5 and 27, respectively, and the series has a sum 192, then the number of terms in the series is

(1) 18 (3) 14

(2) 11 (4) 12

$$192=\frac{n}{2}(5+27)\Rightarrow 192=\frac{n}{2}(32)$$
$$16n=192\Rightarrow n=12$$

(4)

4. Find the sum of each arithmetic series described or shown below.

(a) The sum of the first 100 even, natural numbers.

$$S_{100}=2+4+6+\cdots+200$$
$$=\frac{100}{2}(2+200)=50(202)$$
$$=10,100$$

(b) The sum of multiples of five from 10 to 75, inclusive.

$$75=10+5(n-1)\Rightarrow 75=10+5n-5$$
$$75=5n+5\Rightarrow 5n=70\Rightarrow n=14$$
$$S_{14}=\frac{14}{2}(10+75)=7(85)=595$$

(c) A series whose first two terms are -12 and -8, respectively, and whose last term is 124.

From the first two terms we can see that the common difference is $d=4\Rightarrow$

$$124=-12+4(n-1)\Rightarrow 124=-12+4n-4$$
$$124=4n-16\Rightarrow 4n=140\Rightarrow n=35 \text{ terms}$$
$$S_{35}=\frac{35}{2}(-12+124)=1,960$$

(d) A series of 20 terms whose last term is equal to 97 and whose common difference is five.

$$a_{20}=a_1+19d\Rightarrow 97=a_1+19(5)$$
$$97=a_1+95\Rightarrow a_1=2$$
$$S_{20}=\frac{20}{2}(2+97)=990$$

5. For an arithmetic series that sums to 1,485, it is known that the first term equals 6 and the last term equals 93. *Algebraically* determine the number of terms summed in this series.

$$1485=\frac{n}{2}(6+93)\Rightarrow 1485=\frac{n}{2}(99)\Rightarrow 99n=2970\Rightarrow n=30$$

APPLICATIONS

6. Arlington High School recently installed a new black-box theatre for local productions. They only had room for 14 rows of seats, where the number of seats in each row constitutes an arithmetic sequence starting with eight seats and increasing by two seats per row thereafter. How many seats are in the new black-box theatre? Show the calculations that lead to your answer.

$$a_{14}=a_1+13d=8+13(2)=34 \text{ seats in last row}$$

$$S_{14}=\frac{14}{2}(8+34)=7(42)=294 \text{ total seats}$$

7. Simeon starts a retirement account where he will place \$50 into the account on the first month and increasing his deposit by \$5 per month each month after. If he saves this way for the next 20 years, how much will the account contain in principal?

240 months

50, 55, 60, ...

a_1

a_n

$$a_{240}=a_1+239d=50+239(5)=\$1245 \text{ in the last month}$$

$$S_{240}=\frac{240}{2}(50+1245)=120(1295)=\$155,400$$

8. The distance an object falls per second while only under the influence of gravity forms an arithmetic sequence with it falling 16 feet in the first second, 48 feet in the second, 80 feet in the third, etcetera. What is the total distance an object will fall in 10 seconds? Show the work that leads to your answer.

This is an arithmetic sequence whose terms are separated by a common difference of 32. So, we can find the last or 10th term:

$$d_{10}=d_1+9(32)=16+9(32)=304$$

$$\sum_{i=1}^{10} d_i=\frac{10}{2}(16+304)=5(320)=1,600 \text{ ft}$$

9. A large grandfather clock strikes its bell once at 1:00, twice at 2:00, three times at 3:00, etcetera. What is the total number of times the bell will be struck in a day? Use an arithmetic series to help solve the problem and show how you arrived at your answer.

We can start adding up the number of times the clock strikes:

$$1+2+3+\cdots+12+1+2+3+\cdots 12$$

do this twice

$$1+2+3+\cdots+12=\frac{12}{2}(1+12)=6(13)=78$$

$$78+78=156 \text{ bell strikes per day}$$

we multiply by every time

A sequence is **geometric** if the ratio of consecutive terms are the same (multiply by a nonzero constant). A geometric sequence comes in the form $a_1, a_1r, a_1r^2, a_1r^3, \ldots$ where a_1 is the first term and r is the common ratio. The nth term of a geometric sequence is $a_n = a_1 r^{n-1}$. If the geometric sequence is finite, the sum of the first n terms or the nth partial sum is $S_n = \frac{a_1(1-r^n)}{1-r}$

sum of the terms

specific term

1. Determine if the following sequences are geometric.

(a) 5, 10, 20, 40 ,…

$\frac{10}{5} = 2$.

$\frac{20}{10} = 2$.

$\frac{40}{20} = 2$.

yes geometric

(b) 1, 2, 6, 24, …

$\frac{2}{1} = 2$

$\frac{6}{2} = 3$

Different therefore NOT Geometric

2.. Write a rule for the nth term of the geometric sequence $6, -3, \frac{3}{2}, -\frac{3}{4}, \ldots$ and find a_6.

specific term

a_1

sixth term

$a_n = a_1 r^{n-1}$

$\frac{-3}{6} = -\frac{1}{2} = r$

$\frac{3/2}{-3} = -\frac{1}{2}$

$a_1 = 6$

$r = -\frac{1}{2}$

✳ Find r by dividing any term by the one before it

$a_n = 6\left(-\frac{1}{2}\right)^{n-1}$

$a_6 = 6\left(-\frac{1}{2}\right)^{6-1}$

$\boxed{a_6 = \frac{-3}{16}}$

specific term geometric

3. Find the 7th term of the sequence 2, 6, 18, 54, …

×3 ×3 ×3

$a_n = a_1 \cdot r^{n-1}$

6, 7, 8, 9, 10, 11

$a_1 = 2$

$n = 7$

$r = 3$

$a_7 = 2 \cdot 3^{7-1}$

$= 2 \cdot 3^6$

$a_7 = \boxed{1458}$

4. Given that $a_1 = 5$ and $a_2 = 15$ are the first two terms of a geometric sequence, determine the values of a_3 and a_{10}. Show the calculations that lead to your answers.

$\underset{a_1}{5}$ $\underset{a_2}{15}$ ×3 $\underset{a_3}{45}$ $\underset{a_4}{\_\_}$ __ __ __ __ __ $\underset{a_{10}}{98415}$

$r = \frac{15}{5} = 3$

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_3 = 45}$

$\boxed{a_{10} = 98415}$

$a_{10} = 5 \cdot (3)^{10-1} \rightarrow 5 \cdot 3^9 \rightarrow 98415$

5. Find the sum of the first 8 terms of the sequence -5, 15, -45, 135, …

$S_n = \frac{a_1(1-r^n)}{1-r}$

$\frac{15}{-5} = -3$

$\frac{-45}{15} = -3$

$\frac{135}{-45} = -3$

geo with r value of -3

$r = -3$

$n = 8$

$a_1 = -5$

$S_8 = \frac{-5(1-(-3)^8)}{1-(-3)}$

$\boxed{\text{Sum} = 8200}$

6. Write the first five terms of the geometric sequence whose first term is $\frac{1}{5}$ and whose common ratio is $-\frac{1}{5}$.

$= a_1$

$= r$

$\frac{1}{5}$ $\quad -\frac{1}{25}$ $\quad \frac{1}{125}$ $\quad \frac{-1}{625}$ $\quad \frac{1}{3125}$

$\left(\frac{1}{5}\right)\left(\frac{-1}{5}\right)$ $\quad \left(\frac{-1}{25}\right)\left(\frac{-1}{5}\right)$ $\quad \frac{1}{125}\left(\frac{-1}{5}\right)$ $\quad \frac{-1}{625}\left(\frac{-1}{5}\right)$

specific term

7. Find the 15th term of the geometric sequence whose 1st term is 20 and whose common ratio is 2.

r a_1

$$a_n = a_1 \cdot r^{n-1}$$

$a_1 = 20$
$r = 2$
$n = 15$

$$a_{15} = 20 \cdot 2^{15-1}$$
$$= 20(2)^{14}$$
$$= \boxed{327680}$$

8. The first term of a geometric sequence is 8, and the second term is 4. Find the fifth term.

8 4 __ __ $\frac{1}{2}$

$\times\frac{1}{2}$

$r = \frac{4}{8} = \frac{1}{2}$

$$a_5 = 8\left(\tfrac{1}{2}\right)^4$$
$$= 8\left(\tfrac{1}{16}\right)$$
$$= \frac{1}{2}$$

9. The 4th term of a geometric sequence is 125 and the 10th term is $\frac{125}{64}$. Find the 14th term.

Start End

125 (a_1) ... $\frac{125}{64}$ ... a_{11}

6 spots 4 spots

$$\sqrt[6]{\frac{\frac{125}{64}}{125}} = r = \frac{1}{2}$$

$$= 125\left(\tfrac{1}{2}\right)^{10} = \frac{125}{1024}$$

$$= \frac{125}{64}\left(\tfrac{1}{2}\right)^{4} = \frac{125}{1024}$$

10. Find the 7th term of the geometric sequence whose third term is $\frac{16}{3}$ and whose fifth term is $\frac{64}{27}$.

11. Which term of the geometric sequence 2, 6, 18, … is 118,098?

12. The second and fifth terms of a geometric sequence are 10 and 1250 respectively. Is 31,250 a term of this sequence? If so, which term is it?

13. Find the sum of each:

(a) 1, 3, 9, …, 2187

(b) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots -\frac{1}{512}$

14. In a geometric sequence, it is known that $a_1 = -1$ and $a_4 = 64$. The value of a_{10} is

(1) −65,536 (3) 512

(2) 262,144 (4) −4096

15. Generate the next **three** terms of each geometric sequence defined below.

(a) $a_1 = -8$ with $r = -1$

(b) $a_n = a_{n-1} \cdot \frac{3}{2}$ and $a_1 = 16$

(c) $f(n) = f(n-1) \cdot -2$ and $f(1) = 5$

16. Generate the next three terms of the geometric sequences given below.

(a) $a_1 = 4$ and $r = 2$

(b) $f(n) = f(n-1) \cdot \frac{1}{3}$ with $f(1) = 9$

(c) $t_n = t_{n-1} \cdot \sqrt{2}$ with $t_1 = 3\sqrt{2}$